1 INTRODUCTION

A press fit is a process for the assembling of two parts. The parts are pressed together at room temperature by tools (assembly punch and assembly die) using a joining force, which is provided by the assembly press. The inner part (e.g., shaft) is oversized for the space in the outer part (core or housing, for example); therefore, two parts interfere with each other’s occupation of space. Both parts deform to fit together into the assembly and create a normal force. The friction force, which is caused by the normal force, holds the parts together and prevents disassembly during the utilization of the assembly. The selection of the contact dimensions of parts to be assembled determines the tightness of fit, the joining force, and subsequently the disassembly force during use, as explained in 1–5.

Regardless of the simplicity of the press-fit process principle, there is a lack of generality due to the diversity of industrial possibilities in contemporary literature, although its outstanding potential in serial production could be well utilized.1,6,4 In that way, the pre-production analysis of influential process parameters is essential.

The relevant press-fit process research comprises:

• joining materials analysis done by 1,5–8
• geometry analysis done by 1,6,9,10,11
• studies of load-specific applications done by 11
• prediction of stresses and deformations during press-fit processes by 1,6,7,9–13.

For the prediction of stresses during press-fit processes, analytical methods are used by 1,6,10 and finite-element methods are used by 7–9,13.

The idea of the research work presented in this paper is to optimize the press-fit process in the early stage of the development process involving prediction and opti-
mization of the joining force and, consequently, the prediction and minimization of the rejection rate, which makes the approach unique.

First, a case study is presented. Afterwards, the FE model for simulations of the press-fit process is explained and verified by comparing the predicted joining force with the measured one at the assembly press. Next, the development of an empirical model for predicting the joining force using RSM is shown. Afterwards, input press-fit process parameter optimization and the rejection-rate prediction using stochastic Monte Carlo simulation are addressed. In the end, conclusions are drawn, and future work is described.

2 CASE STUDY

For the purpose of this research, pistons of solenoid valves are studied that are mass produced for press-fit assembly by many companies worldwide. The case study is presented in Figure 1.

The core is machined from the material 11SMnPb30, which is widely used due to its good machinability and the easy fragmentation of chips. The shaft is produced from the material CuZn39Pb3, which also possesses excellent machinability. The mechanical properties and $\sigma$-e curves for both materials were obtained via a standard tensile test at room temperature as described in 14 and 15 and are presented in Figure 2 and Table 1.

Table 1: Mechanical properties of materials 11SMnPb30 and CuZn39Pb3

<table>
<thead>
<tr>
<th>Part</th>
<th>Material</th>
<th>$E$ (MPa)</th>
<th>$R_p$ (MPa)</th>
<th>$R_m$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaft</td>
<td>CuZn39Pb3</td>
<td>96000</td>
<td>350</td>
<td>480</td>
</tr>
<tr>
<td>Core</td>
<td>11SMnPb30</td>
<td>211000</td>
<td>530</td>
<td>572</td>
</tr>
</tbody>
</table>

The shaft and the core are chamfered, and the core is designed in such a way that its inner dimension decreases gradually from guide diameter to functional diameter ($D_{core1} > D_{core2}$).

The minimum force of disassembly is defined by the designer of the assembly. In the studied case, a disassembly force higher than 200 N is required ($F_{AMIN} = 200 N$). The force for disassembly can be estimated as equal to the joining force because of the characteristics of friction (at a given normal force and the coefficient of friction, the friction force is equal in all directions).

The maximum joining force, which causes plastic deformation and upsetting of the shaft $F_{AMAX}$, can be calculated by using the following Equation (1):

$$F_{MAX} = \frac{\pi d_{shaft}^2}{4} R_{pshaft}$$

where:

$R_{pshaft} = \text{yield stress of shaft material and } d_{shaft} = \text{diameter of shaft.}$

In general, pressing a non-guided slender shaft into the core (Figure 1) could, under certain circumstances, result in buckling deformation of the shaft and the consequent runout. The critical axial buckling force ($F_{BUCKLING}$) can be analytically determined with Euler’s formula. However, in the presented case, plastic deformation was the only limiting parameter ($F_{AMAX} < F_{BUCKLING}$), excluding the buckling in further steps of the study. In general, the stress state in the core should also be regarded as the limiting parameter for evaluation of the maximum-allowable joining force. However, in the presented case the shaft is machined from much softer material than the core and the stress state in the core is not critical.

In all cases, the joining force $F_A$ is the most important process output that can be used to evaluate the feasibility of the press-fit assembly process. Therefore, in practice during the assembly operation, the joining force $F_A$ is measured, and all products are ejected, where the joining force $F_A$ at the final state of the press fit is not within the prescribed range $F_{AMIN} < F_A < F_{AMAX}$. 

Figure 2: $\sigma$-e curves of materials 11SMnPb30 and CuZn39Pb3

Figure 1: Press-fit process
2.1 FE analysis

An FE model was set up for the investigation of how the input parameter influences the joining force \( F_A \) (Figure 3). The model was defined as static (2D) axisymmetric. This model is preferred from the computational times’ point of view because the studied system is axisymmetrical and inertial forces during assembly or disassembly process can be neglected. For the discretization of both parts, 3-node triangle and 4-node quadrilateral axisymmetric linear elements were used. For both parts, an elastoplastic material model was used. A surface-to-surface contact was defined between the two parts. For this contact pair, the total normal contact force \( F_N \) was calculated at different lengths of engagement \( L \). The necessary assembly force was then calculated with Coulomb’s Law, where the coefficient of friction was approximated as \( \mu = 0.075 \), as suggested in \(^{17}\) for a steel-brass dry contact.

Calculated joining force for selected combination of input dimensions \( d_{\text{shaft}} = 1.993 \text{ mm}, D_{\text{core1}} = 2 \text{ mm} \) and \( D_{\text{core2}} = 1.965 \text{ mm} \) is presented in Figure 4. The final engagement length \( L_{\text{MAX}} = 6.8 \text{ mm} \) is achieved when the shaft end reaches the stopping hole. From that point, the required assembly force increases rapidly.

To validate the quality of the FE model, the predicted joining force \( F_A \) was compared to experimental results; 30 samples were produced on the assembly press equipped with the force-measurement sensor. Their input dimensions were the same as those for numerical simulations. The following values joining force \( F_A \) were measured: average = 505 N, minimal = 475 N, maximal = 560 N. The FE model, therefore, predicted 14 % higher joining force \( F_A \), than the average value measured.

Furthermore, the process window of the press-fit process was calculated by using the developed FE model and repeating the FE simulations with several different combinations of \( d_{\text{shaft}} \) and \( D_{\text{core}} \) (Figure 5).

Certain combinations \( d_{\text{shaft}} - D_{\text{core2}} \) are leading to an insufficient joining force \( F_A \) and others are leading to plastic deformation of the shaft \( (F_A > F_{A \text{MAX}}) \), which is calculated by Equation (1)). An acceptable combination of \( d_{\text{shaft}} \) and \( D_{\text{core2}} \) in the middle represents the process window of the studied press-fit process.

The upper part of the process window is unusable in industrial practice (it is impossible to produce the core with \( D_{\text{core2}} > D_{\text{core1}} = 2 \text{ mm} \) with standard cost-effective machining processes) but this does not change/influence the approach for the robust design of press-fit processes proposed in this paper.

Figure 3: FE model

Figure 4: Predicted joining force

Figure 5: The process window for the studied press-fit process
Intuitively, it would be reasonable to set the mentioned control input parameters exactly in the middle of the process window. But the question of how to evaluate and how to minimize the rejection rate (by selecting optimal combination of $d_{\text{shaft}}$, $D_{\text{core1}}$, and $D_{\text{core2}}$) of the studied press-fit process remains.

2.2 Analysis of joining force

To predict how the joining force $F_A$ varies during the press-fit process, the following steps were performed:

Estimation of expected variations of input variables;

Development of empirical model for predicting joining force $F_A$ influenced by $E_{\text{shaft}}$, $R_{p_{\text{shaft}}}$, $\sigma_{\text{strain}}$, $d_{\text{shaft}}$, $E_{\text{core}}$, $R_{p_{\text{core}}}$, $\sigma_{\text{strain}}$, $D_{\text{core1}}$, $D_{\text{core2}}$, and $\mu$.

Calculations of variations of joining force using Monte Carlo method.

2.2.1 Input parameters

Each input parameter of the press-fit process should be considered as a probabilistic variable. In our study, the variations of the input parameters were not actually determined by measurements and experiments, but estimated according to prior experiences. In Table 2, the most relevant input parameters, their nominal values and expected variations are gathered.

Table 2: Nominal values and expected variations of input variables.

<table>
<thead>
<tr>
<th>Input variable</th>
<th>Mean value and expected variation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SHAFT</strong></td>
<td></td>
</tr>
<tr>
<td>Yield stress (MPa)</td>
<td>$R_{p_{\text{shaft}}} = 350\pm40$</td>
</tr>
<tr>
<td>Hardening exponent (1)</td>
<td>$n_{\text{shaft}} = 0.16\pm0.02$</td>
</tr>
<tr>
<td>Diameter of the shaft (mm)</td>
<td>$d_{\text{shaft}} = 1.993\pm0.007$</td>
</tr>
<tr>
<td><strong>CORE</strong></td>
<td></td>
</tr>
<tr>
<td>Yield stress (MPa)</td>
<td>$R_{p_{\text{core}}} = 530\pm50$</td>
</tr>
<tr>
<td>Hardening exponent (1)</td>
<td>$n_{\text{core}} = 0.16\pm0.02$</td>
</tr>
<tr>
<td>Guide diameter of the core (mm)</td>
<td>$D_{\text{core1}} = 2\pm0.012$</td>
</tr>
<tr>
<td>Functional diameter of the core (mm)</td>
<td>$D_{\text{core2}} = 1.965\pm0.012$</td>
</tr>
<tr>
<td><strong>OTHER</strong></td>
<td></td>
</tr>
<tr>
<td>Coefficient of friction</td>
<td>$\mu = 0.075\pm0.02$</td>
</tr>
</tbody>
</table>

The slopes of $\sigma$-$\varepsilon$ curves in the plastic region were approximated using the hardening law $\sigma_1 = \varepsilon^{n_{\text{strain}}}$. In Table 3, hardening exponents for both materials are presented.

Expected variations of the material properties ($R_{p_{\text{shaft}}}$, $n_{\text{shaft}}$, $R_{p_{\text{core}}}$, $n_{\text{core}}$) are based on the data previously gathered in different forming processes. Experimental work, presented in 19 and 20 reports comparable variations of material properties.

The expected variation of the diameter of the shaft $d_{\text{shaft}}$ was selected since the wires with tolerances h8 are commercially available and widely used in various industrial applications. The expected variations of the diameters of the core $D_{\text{core1}}$ and $D_{\text{core2}}$ were selected due to the fact that such tolerances are achievable in state-of-the-art machining operations with reasonable costs. The expected variation of the friction coefficient $\mu$ is also based on previously gathered data in 18.

In the presented work, it was assumed that the variations of all the input variables are normally distributed with standard deviations equal to one quarter of the expected variations specified in Table 2.

A part of the experimental matrix (6 out of 47 runs) can be seen in Table 3. According to the selected design of experiments, FE simulations were run for the prediction of joining force $F_A$ for different setting of input variables (right column of Table 3).

2.2.2 Development of empirical model for predicting joining force $F_A$ using response surface methodology

RSM is a method for the determination of the relationships between several input parameters and one or more output parameters (also termed responses of the studied system) and is further described in 27. Different designs of experiments can be used. We used a three-level Box-Behnken Design. The low and high levels of input variables were selected in such a way as to cover the area of input parameters, which was later used for optimization.

The response function coefficients were determined by a standard method of least squares, which minimizes the sum of the squared deviations of fitted values. It was expected that the behaviour of the forming system is non-linear; therefore, a second-order polynomial function was used. The fitness of the response function has been estimated using the Analysis Of Variance (ANOVA) technique as described in 21. The $R$-squared

Table 3: Experimental design matrix and results of FE simulations

<table>
<thead>
<tr>
<th>Run</th>
<th>$R_{p_{\text{core}}}$</th>
<th>$n_{\text{core}}$</th>
<th>$D_{\text{core1}}$</th>
<th>$D_{\text{core2}}$</th>
<th>$R_{p_{\text{shaft}}}$</th>
<th>$n_{\text{shaft}}$</th>
<th>$d_{\text{shaft}}$</th>
<th>$\mu$</th>
<th>$F_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>480</td>
<td>0.14</td>
<td>1.98</td>
<td>1.945</td>
<td>310</td>
<td>0.18</td>
<td>2.000</td>
<td>0.055</td>
<td>1003</td>
</tr>
<tr>
<td>2</td>
<td>480</td>
<td>0.14</td>
<td>2.02</td>
<td>1.985</td>
<td>390</td>
<td>0.18</td>
<td>1.986</td>
<td>0.055</td>
<td>35</td>
</tr>
<tr>
<td>3</td>
<td>480</td>
<td>0.18</td>
<td>2.02</td>
<td>1.945</td>
<td>310</td>
<td>0.14</td>
<td>2.000</td>
<td>0.095</td>
<td>1385</td>
</tr>
<tr>
<td>4</td>
<td>480</td>
<td>0.18</td>
<td>1.98</td>
<td>1.945</td>
<td>390</td>
<td>0.18</td>
<td>2.000</td>
<td>0.055</td>
<td>997</td>
</tr>
<tr>
<td>5</td>
<td>580</td>
<td>0.14</td>
<td>2.02</td>
<td>1.985</td>
<td>310</td>
<td>0.14</td>
<td>2.000</td>
<td>0.055</td>
<td>1142</td>
</tr>
<tr>
<td>47</td>
<td>530</td>
<td>0.16</td>
<td>2.00</td>
<td>1.93136</td>
<td>350</td>
<td>0.16</td>
<td>1.993</td>
<td>0.075</td>
<td>777</td>
</tr>
</tbody>
</table>
value of the model is 0.9991 and average relative deviation between predicted and calculated values is 2.54 %.

2.2.3 Calculations of variations of joining force using developed empirical model and Monte Carlo method

A Monte Carlo simulation is a method to determine the probabilistic response of complex systems. The principle of this method is to use a random number generator to simulate the variations of the input variables. Once the empirical model for joining force $F_A$ was obtained, using a RSM model, it was possible to use the Monte Carlo techniques to evaluate the variation of the response of the system (joining force $F_A$) due to variations of the input parameters. The predicted variations of joining force $F_A$ for the nominal average values of all input parameters and their expected variations (gathered in Table 2) is presented in the upper part of Figure 6.

In the lower part of Figure 6, the actual distribution of the measured joining force $F_A$ is presented. The measurements were performed for 5 months; 21800 test pieces were produced at this time. The actual dispersion of the joining force $F_A$ is similar to the predicted one.

2.3 Prediction of rejection rate and optimization of press-fit process

The rejection rate can be evaluated from the probability chart for joining force $F_A$. From the upper part of Figure 6, it can be predicted that 98.6 % of the parts produced would be within the required tolerance and the rejection rate would be 1.4 % (due to the plastic deformation of the shaft). As can be seen from the lower part of Figure 6, the measured level of the rejection rate was 1.5 %.

Finally, the developed approach can be used for an optimization of the press-fit process. Assume that the material of shaft and core are selected and that wire for the shaft must be purchased in a standard dimension ($d_{shaft} = 2 \text{ mm} \ h8$). In this case, the two major input variables that can be influenced and optimized are $D_{core1}$ and $D_{core2}$ (which are produced in the machining department of the company). While using an empirical model and Monte Carlo simulations varying of $D_{core1}$ and $D_{core2}$ ($D_{core1}$ varies from 2 mm to 2.01 mm and $D_{core2}$ from 1.965 mm to 1.97 mm), the calculated predicted rejection rate can be easily presented in the 3D graph (Figure 7).

It is shown that by using the optimal combination of input parameters ($D_{core1} = 2.01 \text{ mm}$ and $D_{core2} = 1.97 \text{ mm}$) the rejection rate can be reduced from 1.4 % to 0.2 %.

3 CONCLUSIONS

In the research, the pistons of solenoid valves are studied, which are mass produced for press-fit assembly by many companies worldwide. An FE model was set up for the investigation of how the input parameters influence the joining force $F_A$ (Figure 3). The necessary assembly force was then calculated by Coulomb’s Law, where the coefficient of friction was approximated as $\mu = 0.075$ for a steel-brass dry contact.

Our approach, which was a combination of FE calculation for prediction of normal contact force and empirical calculation of friction force with Coulomb’s Law, predicted a 14 % higher joining force than the average measured value. The results could be further improved...
by repeating the calculations with a 14% lower coefficient of friction $\mu$.

Furthermore, the process window of the press-fit process was determined by repeating the FE simulations with several different combinations of core and shaft diameters.

Afterwards, the variations of the shaft parameters (diameter, yield stress, hardening exponent), core parameters (guide and functional diameter, yield stress, hardening exponent) and friction coefficient were analysed.

Based on the FE simulations, using 47 different input parameter variation results, the empirical model for predicting joining force using RSM (Response Surface Methodology) method was obtained. The average relative deviations between the predicted and calculated values of the joining forces were 2.54%. The model and Monte Carlo technique was used to evaluate the variations of the joining force due to variations of the input parameters.

A Monte Carlo simulation predicted that 98.6% of the parts produced would be within the required tolerance. Consequently, the rejection rate is 1.4% (due to the plastic deformation of the shaft). The actual reject rate (obtained from the testing) was 1.5%. In the study, only a rejection caused by a variation of input process parameters is evaluated. Rejections resulting for other reasons (failure of the tool, the wrong setting of the machine, etc.) were not the subject of the presented study.

Finally, the developed approach was used for the optimization of the press-fit process. It was shown that by using the optimal combination of input parameters, the predicted reject rate can be reduced from 1.4% to 0.2%.

In the future, the cost function should be integrated into the optimization procedure in order to optimize the studied press-fitting processes also from the economic point of view. In some cases, it is reasonable to increase the machining tolerances or use low-cost raw material with higher variations of properties, although the press-fitting process results in a higher rejection rate.

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